Proposal for quantum spin tomography in ferromagnet-normal conductors

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We present a theory for a complete reconstruction of nonlocal spin correlations in ferromagnet-normal conductors. This quantum spin tomography is based on cross-correlation measurements of electric currents into ferromagnetic terminals with controllable magnetization directions. For normal injectors, nonlocal spin correlations are universal and strong. The correlations are suppressed by spin-flip scattering and, for ferromagnetic injectors, by increasing injector polarization.

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I. INTRODUCTION

Spintronics utilizes the electron spin in electronics applications and is an important subfield of condensed-matter physics. It is possible to create metallic or semiconducting hybrid ferromagnet-normal conductor systems smaller than the spin-flip length,[1,2] yet semiclassically large. Topics of current interest focus on the average nonequilibrium spin accumulation and dynamics. These subjects are, e.g., spin injection, precession, and relaxation,[1–3] spin Hall effects,[4] current-induced magnetization excitations,[5] the reciprocal magnetization dynamics-induced spin pumping,[6] spin-based transistors,[7] and ferromagnet-superconductor heterostructures.[8]

The Pauli exclusion principle causes spin correlations. The correlations between injected spins in ferromagnet-normal conductor systems have received much less attention. In two-terminal junctions, current correlations have been investigated in few-level quantum dots[9] as well as semiclassically large systems.[10,11] The prime targets have been noise due to spin-flip scattering and the super- or sub-Poissonian nature of the autocorrelations.

In multiterminal junctions, current cross correlations allow investigations of nonlocal spin-transport properties. Of main interest has been the sign of the cross correlations, studied in quantum dots,[12] diffusive[13] and superconducting[14] systems, and chaotic cavities.[15] Moreover, in the context of entanglement of itinerant spins, works on few mode[16] and recently also semiclassical[17,18] conductors considered nonlocal detection schemes with cross correlations between currents in noncollinear ferromagnetic terminals.

A fundamental and important question which has not been addressed is if known nonlocal spin injection and detection schemes[1,3] can be extended to identify nonlocal spin correlations. Imagine spins injected into a normal conductor and detected at two different spatial locations by ferromagnetic terminals. What are the nonlocal spatial correlations between the spins? Is it possible to completely characterize the correlations by experimentally accessible electrical current correlations? We provide answers to these questions for semiclassical systems: (i) nonlocal spin correlations are strong, and for normal injectors, universal and (ii) spin correlations can be reconstructed by a sequence of measurements of correlations of currents at ferromagnetic detectors with controllable magnetization directions, a quantum spin tomography.

We consider a semiclassically large, normal (metal or semi) conductor connected to a normal or ferromagnetic injector, biased at a voltage V, and two spatially separated detectors, A and B, see Fig. 1. Detector A (B) consists of a normal node coupled to grounded ferromagnetic terminals $A1$ and $A2$ ($B1$ and $B2$) via tunnel contacts with conductances G_{A1} and G_{A2} (G_{B1} and G_{B2}). Throughout, conductances are dimensionless and in units of the conductance quantum $2e^2/h$. The detectors A and B probe noninvasively the nonlocal spin correlations.

Let us first summarize and explain our main results (i) and (ii) for the nonlocal correlated spin-transport properties in the device in Fig. 1. First, combining scattering theory and a Boltzmann-Langevin approach we derive an expression for the current correlations

$$S_{AiBj} = \frac{2e^2}{h}\int_0^{eV} dE s_{AiBj}(E) \quad (1)$$

with

$$s_{AiBj} = 4G_{Ai}G_{Bj}\langle(\delta f_A^c + \mathbf{P}_{Ai}\cdot\delta\mathbf{f}_A)(\delta f_B^c + \mathbf{P}_{Bj}\cdot\delta\mathbf{f}_B)\rangle_f, \quad (2)$$

where $\mathbf{P}_{Ai}(\mathbf{P}_{Bj})$ is the polarization of the tunnel contact to terminal Ai (Bj),

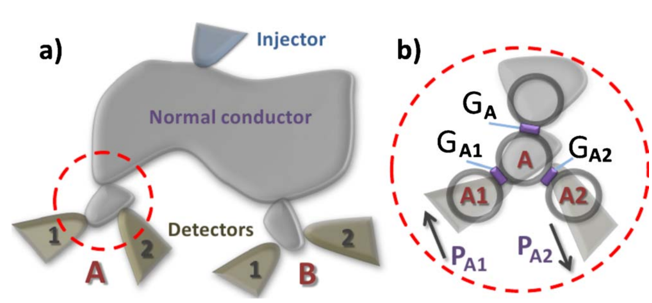

FIG. 1. (Color online) (a) A normal conductor is connected to an injector biased at voltage V and two detector nodes A and B. The node A (B) is coupled to grounded ferromagnetic detector terminals $A1$ and $A2$ ($B1$ and $B2$). (b) Node A is connected to the normal conductor, as well as nodes $A1$ and $A2$ via tunnel conductances G_A, G_{A1}, and G_{A2}, respectively. The polarizations $\mathbf{P}_{A1}$ and $\mathbf{P}_{A2}$ of the contacts to the ferromagnetic terminals are in opposite directions.

$$\delta\hat{f}_{A/B} = \delta f^c_{A/B}\hat{1} + \delta\mathbf{f}_{A/B}\cdot\hat{\boldsymbol{\sigma}}$$

is the fluctuating part of the 2×2 spin-distribution matrix at A/B, $\hat{\boldsymbol{\sigma}}=[\hat{\sigma}_x,\hat{\sigma}_y,\hat{\sigma}_z]$ is a vector of Pauli matrices, and $\langle\cdots\rangle_f$ denotes the average over fluctuations. The matrix $\delta\hat{f}_{AB}$, with elements $\delta f^{pq}_{AB}=\langle\delta f^p_A\delta f^q_B\rangle_f$, $p,q\in\{c,x,y,z\}$, is the spin-correlation matrix, describing the irreducible, or exchange, correlations between spins at A and B.

We then show our result (ii): $\delta\hat{f}_{AB}$ can be reconstructed by a sequence of measurements of, e.g., S_{A1B1} with different settings of $\mathbf{P}_{A1}$ and $\mathbf{P}_{B1}$. Importantly, this quantum spin tomography can be performed for arbitrary (finite) magnitudes of the polarizations $|\mathbf{P}_{A1}|$ and $|\mathbf{P}_{B1}|$ and spin-flip scattering in the conductor. Moreover, global spin symmetries limit the number of finite elements of $\delta\hat{f}_{AB}$, allowing for a simplified quantum spin tomography with fewer cross-correlation measurements.

For a normal injector we derive a generic expression for $\delta\hat{f}_{AB}$ with nonzero elements

$$\delta f^{cc}_{AB} = \delta f^0_{AB}/2, \tag{3}$$

$$\delta f^{xx}_{AB} = \delta f^{yy}_{AB} = \delta f^{zz}_{AB} = \gamma\delta f^0_{AB}/2, \tag{4}$$

where δf^0_{AB} is the equal-spin correlator and γ quantifies the spin coherence in the conductor. $\gamma=1$ for a coherent system, i.e., no spin-flip scattering, and $\gamma=0$ for a system with strong spin-flip relaxation. For a ferromagnetic injector, the correlations depend on the properties of the conductor, as shown below.

Inserting Eqs. (3) and (4) into Eq. (2) gives a cross correlator

$$s_{AiBj} = 2G_{Ai}G_{Bj}\delta f^0_{AB}[1+\gamma\mathbf{P}_{Ai}\cdot\mathbf{P}_{Bj}], \tag{5}$$

depending on the relative orientation of the polarizations $\mathbf{P}_{Ai}$ and $\mathbf{P}_{Bj}$. This together with Eqs. (3) and (4) demonstrate our counterintuitive result (i): *any conductor with a normal injector displays strong and universal nonlocal spin correlations*. We note that for the current cross correlator, similar results have been obtained in particular geometries[16–18] with no spin-flip scattering, $\gamma=1$.

II. QUANTUM SPIN TOMOGRAPHY

We now describe the quantum spin tomography, starting for clarity with the known properties[19] of the average spin-distribution matrix in node A, $\hat{f}_A=f^c_A\hat{1}+\mathbf{f}_A\cdot\hat{\boldsymbol{\sigma}}$, where the real polarization vector $\mathbf{f}_A=[f^x_A,f^y_A,f^z_A]$ with $|\mathbf{f}_A|\leq 1$. The average current is

$$I_{A1} = \frac{e}{h}\int_0^{eV} dE\, i_{A1}(E) \tag{6}$$

with[19]

$$i_{A1} = 2G_{A1}[f^c_A + \mathbf{P}_{A1}\cdot\mathbf{f}_A]. \tag{7}$$

For the quantum spin tomography, we transform the orbital scheme developed in Ref. 20 to the spin degree of freedom and extend it to account for arbitrary detector polarization. Formally, to determine f^c_A and $\mathbf{f}_A$ four independent measurements of the current are needed. The theoretically most convenient set $\{I^{(k)}_{A1}\}$, $k=1-4$ has the polarizations $\mathbf{P}^{(1)}_{A1}/P_{A1}=[0,0,1]$, $\mathbf{P}^{(2)}_{A1}/P_{A1}=[0,0,-1]$, $\mathbf{P}^{(3)}_{A1}/P_{A1}=[1,0,0]$, and $\mathbf{P}^{(4)}_{A1}/P_{A1}=[0,1,0]$, where $P_{A1}=|\mathbf{P}_{A1}|$, but other settings are also feasible. The expression in Eq. (7) then allows writing ($\{k\}=\{c,x,y,z\}$)

$$f^k_A = \frac{\sum_{l=1}^{4} Q^{kl}_{A1}I^{(j)}_{A1}}{4G_{A1}e^2V/h}, \tag{8}$$

where

$$\frac{Q_{A1}}{P_{A1}} = \begin{pmatrix} P^{-1}_{A1} & P^{-1}_{A1} & 0 & 0 \\ -1 & -1 & 2 & 0 \\ -1 & -1 & 0 & 2 \\ 1 & -1 & 0 & 0 \end{pmatrix}. \tag{9}$$

Knowing the polarization P_{A1} and the conductance G_{A1} from independent measurements, the spin-distribution matrix $\hat{f}_A$ is fully reconstructed by current measurements. Importantly, for a normal injector, only f^c_A is nonzero. For a ferromagnetic injector, when the spin-quantization axis along the direction of polarization, only f^c_A and f^z_A are nonzero.

We then turn to the spin-correlation matrix $\delta\hat{f}_{AB}$ with the 16 real elements δf^{pq}_{AB}. This implies that we need 16 independent cross-correlator measurements to determine all elements δf^{pq}_{AB} and reconstruct $\delta\hat{f}_{AB}$. From Eq. (2) we obtain the formal relation between the coefficients δf^{pq}_{AB} and the cross correlators

$$\delta f^{pq}_{AB} = \frac{1}{8G_{A1}G_{B1}Ve^3/h}\sum_{k,l=1}^{4} Q^{pk}_{A1}Q^{ql}_{B1}S^{(k,l)}_{A1B1}, \tag{10}$$

where $S^{(k,l)}_{A1B1}$ is the cross correlator with the detector terminal setting k at $A1$ and l at $B1$. Here Q_{B1} is obtained from Q_{A1} by changing P_{A1} to P_{B1}.

For a normal injector, the requirement[21] of invariance of $\delta\hat{f}_{AB}$ under any global spin rotation means that there is only four nonzero elements δf^{cc}_{AB} and $\delta f^{xx}_{AB}=\delta f^{yy}_{AB}=\delta f^{zz}_{AB}$. For a ferromagnetic injector (defining the spin-quantization axis) invariance of $\delta\hat{f}_{AB}$ under the global rotation $\{|\uparrow\rangle,|\downarrow\rangle\}\to\{e^{i\phi}|\uparrow\rangle,e^{-i\phi}|\downarrow\rangle\}$ yields[22] six nonzero elements δf^{cc}_{AB}, δf^{cz}_{AB}, δf^{zc}_{AB}, δf^{zz}_{AB}, and $\delta f^{xx}_{AB}=\delta f^{yy}_{AB}$.

From Eqs. (9) and (10) the detector polarization settings necessary to determine the nonzero components of $\hat{f}_A$ and $\delta\hat{f}_{AB}$ are found: For a normal injector, only collinear polarizations at A and B are needed for both $\hat{f}_A$ and $\delta\hat{f}_{AB}$. For a ferromagnetic injector, for $\hat{f}_A$ the detector polarizations in addition have to be collinear with the injector one. However, for $\delta\hat{f}_{AB}$ noncollinear polarizations at A and B are necessary, e.g., both along x and z axes, since $\delta f^{zz}_{AB}\neq\delta f^{xx}_{AB}=\delta f^{yy}_{AB}$. Importantly, for an unknown direction of the injector polarization or two (or more) noncollinear ferromagnetic injectors, the full tomographic scheme with detector polarizations along all three axes x, y, and z are required.

III. MODEL AND SCATTERING THEORY

We will now detail our calculations, assumptions, and approximations. In addition to the information given above, the normal conductor in Fig. 1 is connected to detector nodes A and B via tunnel barriers with conductances G_A and G_B. The two ferromagnetic terminals $A1$ and $A2$ ($B1$ and $B2$) have opposite directions of polarization. We assume the limit of low temperature $kT \ll eV$. All conductances are much larger than unity.

It is assumed that the normal conductor consists of diffusive and/or chaotic parts, allowing a semiclassical treatment of the orbital properties. In contrast, spin is treated fully quantum mechanically. Furthermore, scattering is elastic. Following the magnetoelectronic circuit theory of Ref. 19, we discretize the system into nodes connected via tunnel barriers, see Fig. 1. Each node ν, spatially much smaller than the spin-flip length, is characterized by a 2×2 distribution matrix with an average, $\hat{f}_\nu$, and a fluctuating, $\delta\hat{f}_\nu$, part. To ensure that the detectors do not influence the spin properties of the system, we require (i) $G_A \ll G_{A1}+G_{A2}$ and $G_B \ll G_{B1}+G_{B2}$ so that an electron entering, e.g., node A from the conductor is emitted into $A1$ or $A2$ and do not return to the conductor and (ii) $G_{A1}\mathbf{P}_{A1}=-G_{A2}\mathbf{P}_{A2}$ and $G_{B1}\mathbf{P}_{B1}=-G_{B2}\mathbf{P}_{B2}$, which ensures that no spin polarization is induced into the conductor from the ferromagnetic terminals, i.e., the measured spin signal arises from the conductor exclusively and not from the detector circuits.

Deriving Eqs. (2) and (7), we first review[19] the spin information present in the average spectral current $i_{A1}(E)$. In the scattering approach,[23] with no particles incident from terminal $A1$ in the bias window ($0\le E\le eV$), the spectral current is

$$i_{A1} = \sum_{n\sigma} \langle n^\sigma_{A1,n}\rangle, \quad n^\sigma_{A1,n} = b^{\sigma\dagger}_{A1,n} b^\sigma_{A1,n}, \tag{11}$$

where $b^{\sigma\dagger}_{A1,n}$ creates an electron on the ferromagnetic side in the contact between $A1$ and A, in conduction mode n propagating into $A1$ and the energy dependence is suppressed. The spin-quantization axis $\sigma=\uparrow,\downarrow$ is along the direction of $\mathbf{P}_{A1}$. The creation operators $b^{\sigma\dagger}_{A1,n}$ are related to the operators $b^{\tau\dagger}_{Am}$ for electrons on the normal-conductor side, emitted from node A toward $A1$, via the spin-dependent transmission matrix of the normal-ferromagnetic interface t_{A1} with elements $t^{\sigma\tau}_{A1,nm}$. Following Ref. 19, we make the semiclassical approximation that the spin-distribution matrix in node A is independent of mode index, i.e., $\langle b^{\sigma\dagger}_{An} b^{\sigma'}_{Am}\rangle = \hat{f}^{\sigma\sigma'}_A \delta_{nm}$, giving

$$i_{A1} = \sum_{\sigma\tau} \hat{\mathcal{T}}^{\tau\sigma}_{A1} \hat{f}^{\sigma\tau}_A = G_{A1}\,\mathrm{tr}\{(\hat{1}+\mathbf{P}_{A1}\cdot\hat{\boldsymbol{\sigma}})\hat{f}_A\}. \tag{12}$$

Here[19] $\hat{\mathcal{T}}_{A1}=\Sigma_{nm}(\hat{t}_{A1,nm})^T(\hat{t}_{A1,nm})^* = G_{A1}(\hat{1}+\mathbf{P}_{A1}\cdot\hat{\boldsymbol{\sigma}})$, where the elements of the 2×2 matrix are $(\hat{t}_{A1,nm})_{\sigma\tau}=t^{\sigma\tau}_{A1,nm}$. Equation (12) directly gives Eq. (7). Similar relations hold for the average currents into $A2$, $B1$, and $B2$.

We then turn to the low-frequency correlations between electrical currents in, e.g., terminals $A1$ and $B1$, $S_{A1B1} = \int dt \langle \Delta I_{A1}(0)\Delta I_{B1}(t)\rangle$. Scattering theory[23] gives

$$s_{A1B1} = \sum_{nm,\sigma\tau} [\langle n^\sigma_{A1,n} n^\tau_{B1,m}\rangle - \langle n^\sigma_{A1,n}\rangle\langle n^\tau_{B1,m}\rangle], \tag{13}$$

where $n^\tau_{B1,m}=b^{\tau\dagger}_{B1,m}b^\tau_{B1,m}$ and $b^{\tau\dagger}_{B1,m}$ creates an outgoing electron on the ferromagnetic side, in conduction mode m in $B1$ with spin-quantization axis along the direction of the magnetization $\mathbf{n}_B$. Disregarding terms of second order in $G_A/(G_{A1}+G_{A2})$ or $G_B/(G_{B1}+G_{B2})$, the operators $b^{\sigma\dagger}_{A1,n}$ and $b^{\tau\dagger}_{B1,m}$ are expressed in terms of the operators $b^{\sigma'\dagger}_{Ak}$ and $b^{\tau'\dagger}_{Bl}$ and the scattering amplitudes of the respective normal-ferromagnetic interfaces. Making the semiclassical approximation that the nonlocal irreducible correlator

$$\langle b^{\sigma\dagger}_{An} b^{\sigma'}_{Am} b^{\tau\dagger}_{Bk} b^{\tau'}_{Bl}\rangle - \langle b^{\sigma\dagger}_{An} b^{\sigma'}_{Am}\rangle\langle b^{\tau\dagger}_{Bk} b^{\tau'}_{Bl}\rangle \equiv \delta\hat{f}^{\sigma\sigma',\tau\tau'}_{AB}\delta_{nm}\delta_{kl},$$

we arrive at

$$s_{A1B1} = \sum_{\sigma\sigma'\tau\tau'} \hat{\mathcal{T}}^{\sigma'\sigma}_{A1}\hat{\mathcal{T}}^{\tau'\tau}_{B1}\hat{f}^{\sigma\sigma',\tau\tau'}_{AB} = G_{A1}G_{B1} \times \mathrm{tr}\{[(\hat{1}+\mathbf{P}_{A1}\cdot\hat{\boldsymbol{\sigma}})\otimes(\hat{1}+\mathbf{P}_{B1}\cdot\hat{\boldsymbol{\sigma}})]\delta\hat{f}_{AB}\}, \tag{14}$$

where $\hat{\mathcal{T}}_{B1}$ is obtained by changing all indices A to B in $\hat{\mathcal{T}}_{A1}$ and $\otimes$ is the tensor product. Here we work in the basis $\{|\sigma\tau\rangle\}=[|\uparrow\uparrow\rangle,|\uparrow\downarrow\rangle,|\downarrow\uparrow\rangle,|\downarrow\downarrow\rangle]$, i.e., the matrix elements $(\delta\hat{f}_{AB})_{|\sigma\tau\rangle,|\sigma'\tau'\rangle}=\delta\hat{f}^{\sigma\sigma',\tau\tau'}_{AB}$. As is shown below, the 4×4 spin-correlation matrix $\delta\hat{f}_{AB}=\langle\delta\hat{f}_A\otimes\delta\hat{f}_B\rangle_f$ provides a semiclassical interpretation of $\delta\hat{f}_{AB}$. This means that Eq. (14) directly gives Eq. (2). Moreover, the expressions for the other correlators S_{AiBj} can similarly be given in terms of $\delta\hat{f}_{AB}$. This shows that $\delta\hat{f}_{AB}$ *contains all information about nonlocal spin correlations that can be obtained from cross correlations.*

IV. BOLTZMANN-LANGEVIN APPROACH

To further investigate the properties of $\hat{f}_A$, $\hat{f}_B$, and $\delta\hat{f}_{AB}$ we now turn to the spin-dependent Boltzmann-Langevin approach of Ref. 11. The average part of the distribution matrix $\hat{f}_\nu$ at node ν is determined from the condition of conservation of matrix currents into the node,

$$\sum_\mu \hat{i}_{\nu\mu} = 0,$$

where the following symmetry holds $i_{\nu\mu}=-i_{\mu\nu}$. The 2×2 matrix current between a normal node ν and a ferromagnetic or normal node μ is

$$\hat{i}_{\nu\mu} = (G_{\nu\mu}/2)\{(\hat{1}+\mathbf{P}_\mu\cdot\hat{\boldsymbol{\sigma}}),(\hat{f}_\nu-\hat{f}_\mu)\} \tag{15}$$

with $\{\cdot,\cdot\}$ the anticommutator, $G_{\nu\mu}$ the tunnel conductance between the nodes, and $\mathbf{P}_\mu$ the polarization vector of node μ ($\mathbf{P}_\mu=0$ for a normal node). The distribution matrices for normal and ferromagnetic terminal nodes are $\hat{1}$ for biased terminals and 0 for grounded. This allows us to calculate the distribution matrices of all nodes.

For the fluctuating part of the distribution matrix, we first note that the total fluctuations of the matrix current $\Delta\hat{i}_{\nu\mu}$ flowing between two nodes ν and μ is a sum of the bare fluctuations $\delta\hat{i}_{\nu\mu}$ and $\delta\hat{\mathcal{I}}_{\nu\mu}$ due to the fluctuating distribution matrices. For ν normal and μ normal or ferromagnetic

$$\delta\hat{\mathcal{I}}_{\nu\mu} = (G_{\nu\mu}/2)\{(\hat{1} + \mathbf{P}_\mu \cdot \hat{\boldsymbol{\sigma}}),(\delta\hat{f}_\nu - \delta\hat{f}_\mu)\}.$$

The requirement of matrix current-fluctuation conservation $\Sigma_\mu \Delta\hat{i}_{\nu\mu}=0$ then gives $\delta\hat{f}_\nu$ in terms of $\delta\hat{i}_{\nu\mu}$. The bare fluctuations $\delta\hat{i}_{\nu\mu}$ at different contacts are uncorrelated while for ν,μ normal

$$\langle \delta\hat{i}_{\nu\mu} \otimes \delta\hat{i}_{\nu\mu}\rangle_f = \frac{G_{\nu\mu}}{2}[\hat{f}_\nu \otimes (\hat{1} - \hat{f}_\mu) + \hat{f}_\mu \otimes (\hat{1} - \hat{f}_\nu)]\hat{W} + \text{H.c.}, \tag{16}$$

where H.c. denotes hermitian conjugate and the permutation matrix $\hat{W}$ has nonzero elements $W_{11}=W_{23}=W_{32}=W_{44}=1$. For ν normal and μ ferromagnetic we instead have

$$\langle \delta\hat{i}_{\nu\mu} \otimes \delta\hat{i}_{\nu\mu}\rangle_f = \frac{G_{\nu\mu}}{2}\{([\hat{1} + \mathbf{P}_\mu \cdot \hat{\boldsymbol{\sigma}}]f_\mu) \otimes (\hat{1} - \hat{f}_\nu) + ([\hat{1} + \mathbf{P}_\mu \cdot \hat{\boldsymbol{\sigma}}][1 - f_\mu]) \otimes \hat{f}_\nu\}\hat{W} + \text{H.c.} \tag{17}$$

Here we used that ferromagnetic (i.e., terminal) distributions do not fluctuate. From these relations any electrical current correlator $\langle \Delta i_\nu \Delta i_\mu \rangle_f$, with $\Delta i_\nu = \text{tr}[\Delta\hat{i}_\nu]$, can be obtained.

Spin-flip scattering is taken into account on the level of the relaxation-time approximation. This amounts to coupling each node n to a spin-flip node $\varphi\nu$ with a tunnel contact with conductance $G_{\varphi\nu} \propto 1/\tau_{\varphi\nu}$, with $\tau_{\varphi\nu}$ the spin-flip time of the node, and requiring conservation of *electrical* current and current fluctuations into the spin-flip node. Here we give the universal results of the calculation, i.e., we consider an arbitrary normal conductor with any amount of (spatially dependent) spin-flip scattering, the details of the calculations are given elsewhere. First, by comparing the obtained expression for the spectral cross correlators $s_{AiBj}=\langle \Delta i_{A1} \Delta i_{B1}\rangle_f$ with Eq. (14) we conclude that $\delta\hat{f}_{AB}=\langle \delta\hat{f}_A \otimes \delta\hat{f}_B\rangle_f$, discussed above. Second, for normal injectors, we find the generic form

$$\delta\hat{f}_{AB} = (\delta f^0_{AB}/2)[(1-\gamma)\hat{1} \otimes \hat{1} + 2\gamma\hat{W}]. \tag{18}$$

This is just the result in Eqs. (3) and (4).

V. SINGLE-NODE CONDUCTOR

Further insight is obtained by calculating the properties of the simplest possible conductor, a single node.[2] For a normal injector we find the distribution function at, e.g., A as $\hat{f}_A = G_A/(G_{A1}+G_{A2}) f\hat{1}$ with $f=G/(G+G_A+G_B)$ the distribution function of the conductor node and G the injector-conductor node conductance. This is independent of spin-flip scattering. For the spin-correlation matrix we get the result in Eq. (18) with

$$\delta f^0_{AB} = \frac{G^2 G_A G_B}{(G+G_A+G_B)^3 (G_{A1}+G_{A2})(G_{B1}+G_{B2})} \tag{19}$$

and $\gamma=[1+\tau/\tau_\varphi]^{-1}$ with $\tau/\tau_\varphi = G_\varphi/(G_A+G_B+G)$ the ratio of spin flip and dwell times in the central node. To be explicit, the experimentally accessible current cross correlation is thus given by

$$S_{A1B1} = \frac{4e^3 V}{h} G_{A1} G_{B1} \delta f^0_{AB}[1 + \gamma \mathbf{P}_{A1} \cdot \mathbf{P}_{B1}]. \tag{20}$$

For a ferromagnetic injector with polarization $\mathbf{P}_I$ the spin-distribution matrix at, e.g., A has two nonzero components $(\hat{f}_A)^{\uparrow\uparrow} \equiv f_A^\uparrow$ and $(\hat{f}_A)^{\downarrow\downarrow} \equiv f_A^\downarrow$ with $f_A^{\uparrow,\downarrow} = f[(1 \pm P_I)(1 \mp P_I f) + \tau_\varphi/\tau]/[(1-P_I^2 f^2)+\tau_\varphi/\tau]$ with $P_I = |\mathbf{P}_I|$. For the spin-correlation matrix, the full expression, including spin-flip scattering, becomes very lengthy and we only present the result for $\gamma=1$. This is $\delta f^{cc}_{AB} = \delta f^{zz}_{AB} = \delta f^0_{AB}(c_+ + c_-)/2$, $\delta f^{cz}_{AB} = \delta f^{zc}_{AB} = \delta\delta f^0_{AB}(c_+ - c_-)/2$, and $\delta f^{xx}_{AB} = \delta f^{yy}_{AB} = \delta f^0_{AB} c_0/2$ with $c_\pm = (1 \pm P_I)^2/(1 \pm P_I f)^3$ and $c_0 = (1-P_I^2)/[1-(P_I f)^2]$. Inserting this into Eq. (14) we get the total-current cross correlator

$$S_{A1B1} = \frac{4e^3 V}{h} G_{A1} G_{B1} \delta f^0_{AB}(c_I + c_0[1 + \mathbf{P}_{A1} \cdot \mathbf{P}_{B1}]) \tag{21}$$

with

$$c_I = (1 + P^z_{A1} P^z_{B1})\frac{c_+ + c_- - 2c_0}{2} + (P^z_{A1} + P^z_{B1})\frac{c_+ - c_-}{2}.$$

This clearly demonstrates that while a ferromagnetic injector leads to a polarization of the conductor, it suppresses the spin correlations.

VI. CONCLUSIONS

In conclusion we have presented a scheme for quantum state tomography of nonlocal spin correlations in normal-ferromagnetic conductors. This quantum spin tomography is performed by measuring cross correlations of electrical currents at ferromagnetic terminals with controllable polarization. Nonlocal correlations are generically strong but suppressed by spin-flip scattering and ferromagnetic injectors. All ingredients of our proposal are accessible with present day technology, making an experimental test of our predictions feasible.

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[1] Y. Ohno, D. K. Young, B. Beschoten, F. Matsukura, H. Ohno, and D. D. Awschalom, Nature (London) **402**, 790 (1999); R. Fiederling, M. Keim, G. Reuscher, W. Ossau, G. Schmidt, A. Waag, and L. W. Molenkamp, *ibid.* **402**, 787 (1999); F. J. Jedema, A. T. Filip, and B. J. van Wees, *ibid.* **410**, 345 (2001); F. J. Jedema, H. B. Heersche, A. T. Filip, J. J. A. Baselmans, and B. J. van Wees, *ibid.* **416**, 713 (2002).

[2] M. Zaffalon and B. J. van Wees, Phys. Rev. Lett. **91**, 186601 (2003).

[3] M. Johnson and R. H. Silsbee, Phys. Rev. B **37**, 5312 (1988); **37**, 5326 (1988).

[4] H.-A. Engel, E. I. Rashba, and B. I. Halperin, Theory of Spin Hall Effects in Semiconductors, in *Handbook of Magnetism and Advanced Magnetic Materials*, edited by H. Kronmüller and S. Parkin, (John Wiley & Sons Ltd., Chichester, UK 2007), pp. 2858.

[5] D. C. Ralph and M. D. Stiles, J. Magn. Magn. Mater. **320**, 1190 (2008).

[6] Y. Tserkovnyak, A. Brataas, G. Bauer, and B. Halperin, Rev. Mod. Phys. **77**, 1375 (2005).

[7] S. Datta and B. Das, Appl. Phys. Lett. **56**, 665 (1990); K. Ono, H. Shimada, S. Kobayashi, and Y. Ootuka, J. Phys. Soc. Jpn. **65**, 3449 (1996); G. E. W. Bauer, A. Brataas, Y. Tserkovnyak, and B. J. van Wees, Appl. Phys. Lett. **82**, 3928 (2003).

[8] F. S. Bergeret, A. F. Volkov, and K. B. Efetov, Rev. Mod. Phys. **77**, 1321 (2005).

[9] B. R. Bułka, J. Martinek, G. Michalek, and J. Barnas, Phys. Rev. B **60**, 12246 (1999); R. López and D. Sanchez, Phys. Rev. Lett. **90**, 116602 (2003); F. M. Souza, A. P. Jauho, and J. C. Egues, Phys. Rev. B **78**, 155303 (2008).

[10] E. R. Nowak, M. B. Weissman, and S. S. S. Parkin, Appl. Phys. Lett. **74**, 600 (1999); E. G. Mishchenko, Phys. Rev. B **68**, 100409(R) (2003); A. Lamacraft, *ibid.* **69**, 081301(R) (2004); E. G. Mishchenko, A. Brataas, and Y. Tserkovnyak, *ibid.* **69**, 073305 (2004); R. Guerrero, F. G. Aliev, Y. Tserkovnyak, T. S. Santos, and J. S. Moodera, Phys. Rev. Lett. **97**, 266602 (2006).

[11] Y. Tserkovnyak and A. Brataas, Phys. Rev. B **64**, 214402 (2001).

[12] A. Cottet, W. Belzig, and C. Bruder, Phys. Rev. Lett. **92**, 206801 (2004); Phys. Rev. B **70**, 115315 (2004); O. Sauret and D. Feinberg, Phys. Rev. Lett. **92**, 106601 (2004); D. Sánchez and R. Lopez, Phys. Rev. B **71**, 035315 (2005).

[13] W. Belzig and M. Zareyan, Phys. Rev. B **69**, 140407(R) (2004); M. Zareyan and W. Belzig, *ibid.* **71**, 184403 (2005).

[14] F. Taddei and R. Fazio, Phys. Rev. B **65**, 134522 (2002).

[15] D. Sánchez, R. Lopez, P. Samuelsson, and M. Büttiker, Phys. Rev. B **68**, 214501 (2003).

[16] S. Kawabata, J. Phys. Soc. Jpn. **70**, 1210 (2001); N. M. Chtchelkatchev, G. Blatter, G. B. Lesovik, and T. Martin, Phys. Rev. B **66**, 161320(R) (2002).

[17] A. Di Lorenzo and Yu. V. Nazarov, Phys. Rev. Lett. **94**, 210601 (2005).

[18] J. P. Morten, D. Huertas-Hernando, W. Belzig, and A. Brataas, EPL **81**, 40002 (2008).

[19] A. Brataas, Yu. V. Nazarov, and G. E. W. Bauer, Phys. Rev. Lett. **84**, 2481 (2000); Eur. Phys. J. B **22**, 99 (2001).

[20] P. Samuelsson and M. Büttiker, Phys. Rev. B **73**, 041305(R) (2006).

[21] B. Michaelis and C. W. J. Beenakker, Phys. Rev. B **73**, 115329 (2006).

[22] X. Wang and P. Zanardi, Phys. Lett. A **301**, 1 (2002).

[23] M. Büttiker, Phys. Rev. B **46**, 12485 (1992); Ya. Blanter and M. Büttiker, Phys. Rep. **336**, 1 (2000).